Geometry Basics

Concepts you must know!
How to find the distance between two points (this is the same as finding the length of a segment).

What is the distance between (-6, 6) and (1, -4)?

- Use the Pythagorean Theorem to find the missing side.
  - $a^2 + b^2 = c^2$
  - $10^2 + 7^2 = c^2$
  - $100 + 49 = c^2$
  - $149 = c^2$

$$\sqrt{149} = c$$

$\sqrt{149}$ is between 12 and 13.

Watch this video to review how to find the distance between two points.
How to find the midpoint of a segment

In order to find the midpoint of segment $AC$, first find the slope from $A$ to $C$.

Slope $= \frac{\text{rise}}{\text{run}} = \frac{\text{up 2}}{\text{right 6}}$

Since we want to find the point in the middle, divide the rise and the run by two, to find the new slope that will take you to the midpoint.

Slope to find midpoint $= \frac{\text{up 1}}{\text{right 3}}$

The midpoint of $AC$ is at $(0,2)$. 
This transformation could be described in the following three ways:

Using the rule (formula): \((x + 9, y - 4)\)

In words: translate the shape 9 units to the right and 4 units down.

Using translation vector \(\langle 9, -4 \rangle\)

Watch video explaining this translation.
To reflect a point over the y axis, measure the distance from the point to the y axis and find the point on the other side of the y axis that is located that same distance from the y axis.

So point C is 5 units to the right of the y axis, notice that its image, C' is 5 units to the left of the y axis.

Video explaining reflection over the y axis or over the x axis
Reflection over the x axis

To reflect over the x axis, find the distance from the point to the x axis and count that same distance on the other side of the x axis to find the location of the image.

As you can see point B is 4 units above the x axis, and its image, B' is also 4 units from the x axis but under it.

Video explaining reflecting over the y axis or over the x axis.
When a point is reflected over a line, the line of reflection is the **perpendicular bisector** of the segment connecting the preimage (point P) to the image (point P').

So, $\overline{PM} \cong \overline{MP'}$

And right angles are formed at the intersection.
90° CCW Rotation

Apply the formula to each point

\[(x, y)\]
\[(-y, x)\]

The formula shows that the x value becomes the new y, and the opposite of the y becomes the new x value.

So for example, the image of point C(4,5) is C' (-5,4)

Video explaining how to rotate 90 CCW
180° Rotation

Apply the 90° rotation formula to each point twice. The x value becomes the new y, and the opposite of the y becomes the new x value.

(\(x, y\)) (2, 5)

(\(-y, x\)) (5, 2)

(\(-x, -y\)) (2, -5)

So the image of point C(2,5) is C' (2, -5)
Types of Angles

- You can classify angles according to their measures.

- Acute angle: $0 < x < 90$
- Right angle: $x = 90$
- Obtuse angle: $90 < x < 180$
- Straight angle: $x = 180$

This symbol indicates a right angle.
Linear pair angles are adjacent angles (next to each other) that together form a straight angle.

These two angles are supplementary (their sum is 180°).

So:

\[ m\angle 2 + m\angle 1 = 180° \]

Video explaining linear pair and vertical angles
Vertical angles

When two lines intersect, four angles are formed.

The two angles that are opposite to each other are called vertical angles and they measure the same.

So,

∠1 = ∠3

And

∠2 = ∠4

Video explaining linear pair and vertical angles
When two parallel lines are intersected by a transversal, the angles formed are equal.

Although these angles have specific names, the most important fact to know is that all the acute angles will be equal and all the obtuse angles will be equal.

In the diagram you can see that all the acute angles measure $x$ and all the obtuse angles measure $180 - x$.

Video explaining angles formed by parallel lines and a transversal
To graph an equation of the form $y = mx + b$:

1) graph the $y$ intercept, in this example the $y$ intercept is 2, so put a point at 2 on the $y$ axis

2) Find other points by using the slope, in this case you find them by going up 2, to the right 3 or by going down 2 and to the left 3.

Video explaining how to graph $y = mx + b$
If you know a point on a line and the slope of the line, you can find the equation of the line by using the point-slope formula.

<table>
<thead>
<tr>
<th>Point</th>
<th>Slope</th>
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<tbody>
<tr>
<td>(6, -1)</td>
<td>(m = \frac{1}{2})</td>
</tr>
</tbody>
</table>

\[
y - y_1 = m(x - x_1)
\]

\[
y - (-1) = \frac{1}{2}(x - 6)
\]

\[
y + 1 = \frac{1}{2}x - 3
\]

\[
y + 1 - 1 = \frac{1}{2}x - 3 - 1
\]

\[
y = \frac{1}{2}x - 4
\]
Definitions

**Plane**: a flat surface

**Collinear**: points on the same line

**Coplanar**: points or shapes on the same plane

**Parallel lines**: two *coplanar* lines that never intersect. The symbol for parallel is \(\parallel\)

**Perpendicular lines**: two lines that intersect forming right angles. The symbol for perpendicular is \(\perp\)
Slopes of parallel, perpendicular lines

If two lines are parallel, then their slopes are equal. Here you can see the slope of each line is 2.

If two lines are perpendicular, their slopes are opposite reciprocals. Here you can see the slope of one line is $\frac{2}{3}$ while the other is $-\frac{3}{2}$.
How to find the equation of a line parallel

Video showing how to find equation of parallel or perpendicular line

Write the slope-intercept form of an equation of the line that passes through the point (-2, 5) and is parallel to the graph of the equation $y = -4x + 2$.

What will the slope of the line be if it’s parallel to the line $y = -4x + 2$?

$y - 5 = -4(x - (-2))$

$y - 5 = -4(x + 2)$

$y - 5 = -4x - 8$

$y = -4x - 3$

Parallel lines have the same slope, but different $y$-intercepts.
How to find the equation of a line perpendicular

Video showing how to find equation of parallel or perpendicular line

Write an equation in slope-intercept form for the line that passes through (3, 2) and is perpendicular to the line described by $y = 3x - 1$.

**Step 1** Find the slope of the line.

$y = 3x - 1$  
*The slope is 3.*

The perpendicular line has a slope of $-\frac{1}{3}$

**Step 2** Write the equation in point-slope form.

$y - y_1 = m(x - x_1)$  
*Use point-slope form.*

$y - 2 = -\frac{1}{3}(x - 3)$  
*Substitute $-\frac{1}{3}$ for m, 3 for $x_1$, and 2 for $y_1$.*

**Step 3** Write the equation in slope-intercept form.

$y - 2 = -\frac{1}{3}(x - 3)$  

$y - 2 = -\frac{1}{3}x + 1$  
*Distribute $-\frac{1}{3}$ on the right side.*

$y = -\frac{1}{3}x + 3$  
*Add 2 to both sides.*
Shortcuts to prove two triangles are congruent.

These are the only 5 ways to prove triangles are congruent

<table>
<thead>
<tr>
<th>SSS</th>
<th>SAS</th>
<th>HL (right (\Delta) only)</th>
<th>ASA</th>
<th>AAS</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="SSS Diagram" /></td>
<td><img src="image2.png" alt="SAS Diagram" /></td>
<td><img src="image3.png" alt="HL Diagram" /></td>
<td><img src="image4.png" alt="ASA Diagram" /></td>
<td><img src="image5.png" alt="AAS Diagram" /></td>
</tr>
</tbody>
</table>

All three sides are congruent.

Two sides and the included angle are congruent.

The hypotenuse and one of the legs are congruent.

Two angles and the included side are congruent.

Two angles and a (non-included) side are congruent.

Remember that AAA or SSA (the stinky one) cannot be used to prove that two triangles are congruent.
Triangle Angle Sum

Example:

The sum of the measures of the angles of a triangle is 180°.

\[ m\angle A + m\angle B + m\angle C = 180 \]

\[
\begin{align*}
80^\circ + 65^\circ &= 145^\circ \\
180^\circ - 145^\circ &= 35^\circ
\end{align*}
\]
Isosceles Triangles

An isosceles triangle has two equal sides called the legs. The side that is not equal is called the base.

Example:

\[
\begin{align*}
40^\circ + 40^\circ &= 80^\circ \\
180^\circ - 80^\circ &= 100^\circ
\end{align*}
\]

The base angles of an isosceles triangle are equal.
Relationship between sides and angles

- In every triangle, the longest side is opposite the largest angle.
- In every triangle, the shortest side is opposite the smallest angle.

Example:

Which is the smallest side in the triangle below?

Since angle C is the smallest, the side opposite to it would be the shortest. So the answer is side AB.
Triangle Inequality

• The sum of any two sides must be greater than third side or else the three sides cannot form a triangle.
**Parallelogram Properties**

<table>
<thead>
<tr>
<th>DEFINITION: A <strong>parallelogram</strong> is a quadrilateral with both pairs of opposite sides parallel.</th>
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